OPERATIONS MANAGEMENT

Sixth Edition

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CD TUTORIALS

CD TUTORIAL 1

Statistical Tools for Managers

TUTORIAL OUTLINE

Statistical applications permeate the subject of operations management because so much of decision making depends on probabilities that are based on limited or uncertain information. This tutorial provides a review of several important statistical tools that are useful in many chapters of the text. An understanding of the concepts of probability distributions, expected values, and variances is needed in the study of decision trees, quality control, forecasting, queuing models, work measurement, learning curves, inventory, simulation, project management, and maintenance.

DISCRETE PROBABILITY DISTRIBUTIONS

In this section, we explore the properties of **discrete probability distributions**, that is, distributions in which outcomes are not continuous. When we deal with discrete variables, there is a probability value assigned to each event. These values must be between 0 and 1, and they must sum to 1. Example T1 relates to a sampling of student grades.

EXAMPLE T1

The dean at East Florida University, Nancy Beals, is concerned about the undergraduate statistics training of new MBA students. In a sampling of 100 applicants for next year's MBA class, she asked each student to supply his or her final grade in the course in statistics taken as a sophomore or junior. To translate from letter grades to a numeric score, the dean used the following system:

5. A **4.** B **3.** C **2.** D **1.** F

The responses to this query of the 100 potential students are summarized in the table below. Also shown is the probability for each possible grade outcome. This discrete probability distribution is computed using the relative frequency approach. Probability values are also often shown in graph form as in Figure T1.1.

Probability Distribution for Grades

Grade Letter Outcome	Score Variable (x)	Number of Students Responding	Probability, P(x)
A	5	10	0.1 = 10/100
B	4	20	0.2 = 20/100
C	3	30	0.3 = 30/100
D	2	30	0.3 = 30/100
F	1	10	0.1 = 10/100
		Total = 100	1.0 = 100/100

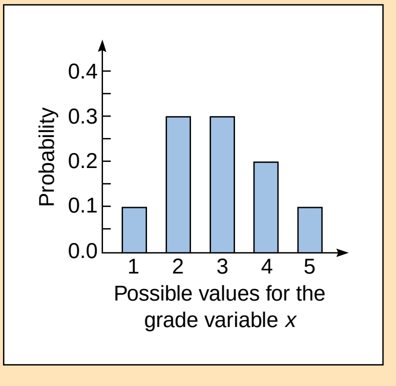

FIGURE T1.1 ■ Probability Function for Grades

This distribution follows the three rules required of all probability distributions:

1. the events are mutually exclusive and collectively exhaustive
2. the individual probability values are between 0 and 1 inclusive
3. the total of the probability values sum to 1

The graph of the probability distribution in Example T1 gives us a picture of its shape. It helps us identify the central tendency of the distribution (called the expected value) and the amount of variability or spread of the distribution (called the variance). Expected value and variance are discussed next.

Expected Value of a Discrete Probability Distribution

Once we have established distribution, the first characteristic we are usually interested in is the "central tendency" or average of the distribution.[1] We computed the **expected value**, a measure of central tendency, as a weighted average of the values of the variable:

$$E(x) = \sum_{i=1}^{n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + \cdots + x_n P(x_n) \qquad \textbf{(T1.1)}$$

where x_i = variable's possible values
$P(x_i)$ = probability of each of the variable's possible values

The expected value of any discrete probability distribution can be computed by: (1) multiplying each possible value of the variable x_i by the probability $P(x_i)$ that outcome will occur, and (2) summing the results, indicated by the summation sign, Σ. Example T2 shows such a calculation.

EXAMPLE T2

Here is how the expected grade value can be computed for the question in Example T1.

$$\begin{aligned} E(x) &= \sum_{i=1}^{5} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5) \\ &= (5)(.1) + (4)(.2) + (3)(.3) + (2)(.3) + (1)(.1) \\ &= 2.9 \end{aligned}$$

The expected grade of 2.9 implies that the mean statistics grade is between D (2) and C (3), and that the average response is closer to a C, which is 3. Looking at Figure T1.1, we see that this is consistent with the shape of the probability function.

Variance of a Discrete Probability Distribution

In addition to the central tendency of a probability distribution, most decision makers are interested in the variability or the spread of the distribution. The **variance** of a probability distribution is a number that reveals the overall spread or dispersion of the distri-

[1] If the data we are dealing with have not been grouped into a probability distribution, the measure of central tendency is called the arithmetic mean, or simply, the average. Here is the mean of the following seven numbers: 10, 12, 18, 6, 4, 5, 15.

$$\text{Arithmetic mean, } \overline{X} = \frac{\sum X}{n} = \frac{10 + 12 + 18 + 6 + 4 + 5 + 15}{7} = 10$$

bution.[2] For a discrete probability distribution, it can be computed using the following equation:

$$\text{Variance} = \sum_{i=1}^{n} (x_i - E(x))^2 P(x_i) \quad \textbf{(T1.2)}$$

where x_i = variable's possible values
$E(x)$ = expected value of the variable
$P(x_i)$ = probability of each possible value of the variable

To compute the preceding variance, the expected value is subtracted from each value of the variable squared, and multiplied by the probability of occurrence of that value. The results are then summed to obtain the variance.

A related measure of dispersion or spread is the **standard deviation**. This quantity is also used in many computations involved with probability distributions. The standard deviation, σ, is just the square root of the variance:

$$\sigma = \sqrt{\text{variance}} \quad \textbf{(T1.3)}$$

Example T3 shows a variance and standard deviation calculation.

EXAMPLE T3

Here is how this procedure is done for the statistics grade survey question:

$$\begin{aligned}\text{Variance} &= \sum_{i=1}^{5} (x_i - E(x))^2 P(x_i)\\ &= (5 - 2.9)^2(.1) + (4 - 2.9)^2(.2) + (3 - 2.9)^2(.3) + (2 - 2.9)^2(.3) + \\ &\quad (1 - 2.9)^2(.1)\\ &= (2.1)^2(.1) + (1.1)^2(.2) + (.1)^2(.3) + (-.9)^2(.3) + (-1.9)^2(.1)\\ &= .441 + .242 + .003 + .243 + .361\\ &= 1.29\end{aligned}$$

The standard deviation for the grade question is

$$\begin{aligned}\sigma &= \sqrt{\text{variance}}\\ &= \sqrt{1.29} = 1.14\end{aligned}$$

[2] Just as the variance of a probability distribution shows the dispersion of the data, so does the variance of ungrouped data, that is, data not formed into a probability distribution. The formula is: Variance = $\Sigma(X - \overline{X})^2/n$. Using the numbers 10, 12, 18, 6, 4, 5, and 15, we find that $\overline{X}$ = 10. Here are the variance computations:

$$\begin{aligned}\text{Variance} &= \frac{(1 - 10)^2 + (12 - 10)^2 + (18 - 10)^2 + (6 - 10)^2 + (4 - 10)^2 + (5 - 10)^2 + (15 - 10)^2}{7}\\ &= \frac{0 + 4 + 64 + 16 + 36 + 25 + 25}{7}\\ &= \frac{170}{7} = 24.28\end{aligned}$$

We should also note that when the data we are looking at represent a *sample* of a whole set of data, we use the term $n - 1$ in the denominator, instead of n, in the variance formula.

CONTINUOUS PROBABILITY DISTRIBUTIONS

There are many examples of continuous variables. The time it takes to finish a project, the number of ounces in a barrel of butter, the high temperature during a given day, the exact length of a given type of lumber, and the weight of a railroad car of coal are all examples of continuous variables. Variables can take on an infinite number of values, so the fundamental probability rules must be modified for continuous variables.

As with discrete probability distributions, the sum of the probability values must equal 1. Because there are an infinite number of values of the variables, however, the probability of *each value* of the variable *must be 0*. If the probability values for the variable values were greater than 0, then the sum would be infinitely large.

The Normal Distribution

One of the most popular and useful continuous probability distributions is the **normal distribution**, which is characterized by a bell-shaped curve. The normal distribution is completely specified when values for the mean, μ, and the standard deviation, σ, are known.

The Area Under the Normal Curve

Because the normal distribution is symmetrical, its midpoint (and highest point) is at the mean. Values of the *x*-axis are then measured in terms of how many standard deviations they are from the mean.

The area under the curve (in a continuous distribution) describes the probability that a variable has a value in the specified interval. The normal distribution requires complex mathematical calculations, but tables that provide areas or probabilities are readily available. For example, Figure T1.2 illustrates three commonly used relationships that have been derived from standard normal tables (a procedure we discuss in a moment). The area from point *a* to point *b* in the first drawing represents the probability, 68%, that the variable will be within 1 standard deviation of the mean. In the middle graph, we see that about 95.4% of the area lies within plus or minus 2 standard deviations of the mean. The third figure shows that 99.73% lies between $\pm 3\sigma$.

Translated into an application, Figure T1.2 implies that if the expected lifetime of an experimental computer chip is $\mu = 100$ days, and if the standard deviation is $\sigma = 15$ days, then we can make the following statements:

1. 68% of the population of computer chips (technically, 68.26%) studied have lives between 85 and 115 days (namely, $\pm 1\sigma$).
2. 95.45% of the chips have lives between 70 and 130 days ($\pm 2\sigma$).
3. 99.73% of the computer chips have lives in the range from 55 to 145 days ($\pm 3\sigma$).
4. Only 16% of the chips have lives greater than 115 days (from first graph, the area to the right of $+1\sigma$).

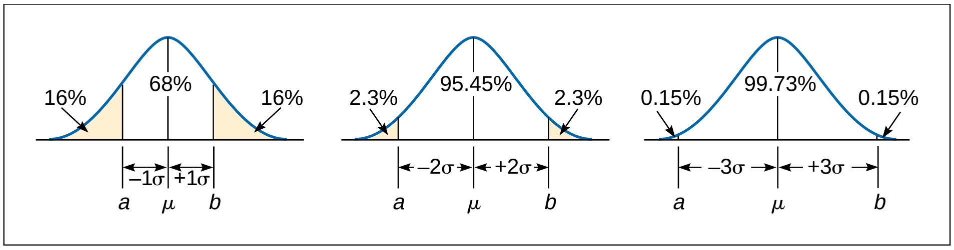

FIGURE T1.2 ■ Three Common Areas Under Normal Curves

Using the Standard Normal Table To use a table to find normal probability values, we follow two steps.

Step 1. Convert the normal distribution to what we call a *standard normal distribution*. A standard normal distribution is one that has a mean of 0 and a standard deviation of 1. All normal tables are designed to handle variables with $\mu = 0$ and $\sigma = 1$. Without a standard normal distribution, a different table would be needed for each pair of μ and σ values. We call the new standard variable z. The value of z for any normal distribution is computed from the equation:

$$z = \frac{x - \mu}{\sigma} \quad \text{(T1.4)}$$

where x = value of the variable we want to measure
μ = mean of the distribution
σ = standard deviation of the distribution
z = number of standard deviations from the mean, μ, to x

For example, if $\mu = 100$, $\sigma = 15$, and we are interested in finding the probability that the variable x is less than 130, then we want $P(x < 130)$.

$$z = \frac{x - \mu}{\sigma} = \frac{130 - 100}{15} = \frac{30}{15} = 2 \text{ standard deviations}$$

This means that the point x is 2.0 standard deviations to the right of the mean. This is shown in Figure T1.3.

Step 2. Look up the probability from a table of normal curve areas. Appendix I in the textbook is such a table of areas for the standard normal distribution. One of the ways it is set up is to provide the area under the curve to the left of any specified value of z.

Let us see how Appendix I can be used. The column on the left lists values of z, with the second decimal place of z appearing in the top row. For example, for a value of $z = 2.00$ as just computed, find 2.0 in the left-hand column and .00 in the top row. In the body of the table, we find that the area sought is .97725, or 97.7%. Thus:

$$P(x < 130) = P(z < 2.00) = 97.7\%$$

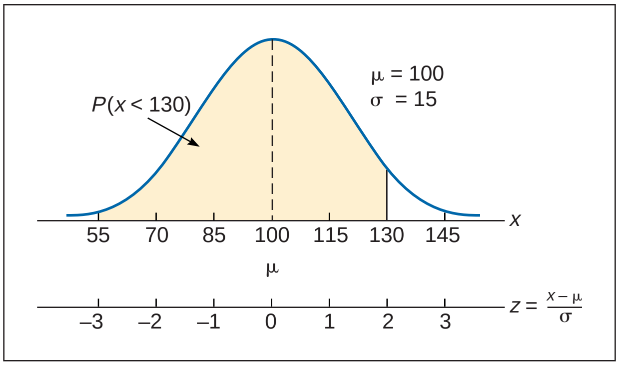

FIGURE T1.3 ■ Normal Distribution Showing the Relationship Between *z* Values and *x* Values

This suggests that if the mean lifetime of a computer chip is 100 days with a standard deviation of 15 days, the probability that the life of a randomly selected chip is less than 130 is 97.7%.

By referring to Figure T1.2, we see that this probability could also have been derived from the middle graph. Note that $1.0 - .977 = .023 = 2.3\%$, which is the area in the right-hand tail of the curve.

Example T4 illustrates the use of the normal distribution further.

EXAMPLE T4

Holden Construction Co. builds primarily three- and four-unit apartment buildings (called triplexes and quadraplexes) for investors, and it is believed that the total construction time in days follows a normal distribution. The mean time to construct a triplex is 100 days, and the standard deviation is 20 days. If the firm finishes this triplex in 75 days or less, it will be awarded a bonus payment of $5,000. What is the probability that Holden will receive the bonus?

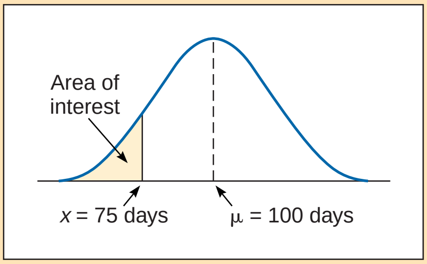

FIGURE T1.4 ■ Probability Holden Will Receive the Bonus by Finishing in 75 Days

Figure T1.4 illustrates the probability we are looking for in the shaded area. The first step is to compute the z value:

$$z = \frac{x - \mu}{\sigma} = \frac{75 - 100}{20} = \frac{-25}{20} = -1.25$$

This z value indicates that 75 days is -1.25 standard deviations to the left of the mean. But the standard normal table is structured to handle only positive z values. To solve this problem, we observe that the curve is symmetric. The probability Holden will finish in *less than 75 days is equivalent* to the probability it will finish in *more than 125 days*. We first find the probability Holden will finish in less than 125 days. That value was .89435. So the probability it will take more than 125 days is

$$P(x < 125) = .89435$$

$$\text{Thus, } P(x > 125) = 1.0 - P(x < 125) = 1.0 - .89435 = .10565$$

Thus, the probability of completing the triplex in 75 days is .10565, or about 10%.

A second example: What is the probability the triplex will take between 110 and 125 days? We see in Figure T1.5 that

$$P(110 < x < 125) = P(x < 125) - P(x < 110)$$

That is, the shaded area in the graph can be computed by finding the probability of completing the building in 125 days or less *minus* the probability of completing it in 110 days or less.

Recall that $P(x < 125 \text{ days})$ is equal to .89435. To find $P(x < 110 \text{ days})$, we follow the two steps developed earlier.

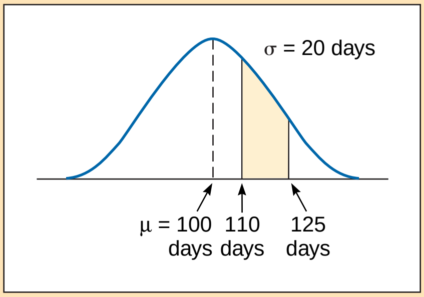

FIGURE T1.5 ■ Probability of Holden Completion Between 110 and 125 Days

1. $z = \frac{x - \mu}{\sigma} - \frac{110 - 100}{20} = \frac{10}{20} = .50$ standard deviation
2. From Appendix I, we see that the area for $z = .50$ is .69146. So the probability the triplex can be completed in less than 110 days is .69146. Finally,

$$P(110 < x < 125) = .89435 - .69146 = .20289$$

The probability that it will take between 110 and 125 days is about 20%.

SUMMARY

The purpose of this tutorial is to assist readers in tackling decision-making issues that involve probabilistic (uncertain) information. A background in statistical tools is quite useful in studying operations management. We examined two types of probability distributions, discrete and continuous. Discrete distributions assign a probability to each specific event. Continuous distributions, such as the normal, describe variables that can take on an infinite number of values. The normal, or bell-shaped, distribution is very widely used in business decision analysis and is referred to throughout this book.

KEY TERMS

Discrete probability distributions *(p. T1-2)*
Expected value *(p. T1-3)*
Variance *(p. T1-3)*
Standard deviation *(p. T1-4)*
Normal distribution *(p. T1-5)*

DISCUSSION QUESTIONS

1. What is the difference between a discrete probability distribution and a continuous probability distribution? Give your own example of each.
2. What is the expected value and what does it measure? How is it computed for a discrete probability distribution?
3. What is the variance and what does it measure? How is it computed for a discrete probability distribution?
4. Name three business processes that can be described by the normal distribution.

PROBLEMS

· **T1.1** Sami Abbasi Health Food stocks five loaves of Vita-Bread. The probability distribution for the sales of Vita-Bread is listed in the following table. How many loaves will Sami sell on the average?

Number of Loaves Sold	Probability
0	.05
1	.15
2	.20
3	.25
4	.20
5	.15

: **T1.2** What are the expected value and variance of the following probability distribution?

Variable, x	Probability
1	.05
2	.05
3	.10
4	.10
5	.15
6	.15
7	.25
8	.15

· **T1.3** Sales for Hobi-cat, a 17-foot catamaran sailboat, have averaged 250 boats per month over the last 5 years with a standard deviation of 25 boats. Assuming that the demand is about the same as past years and follows a normal curve, what is the probability that sales will be less than 280 boats next month?

: **T1.4** Refer to Problem T1.3. What is the probability that sales will be more than 265 boats during the next month? What is the probability that sales will be under 250 boats next month?

: **T1.5** Precision Parts is a job shop that specializes in producing electric motor shafts. The average shaft size for the E300 electric motor is .55 inch, with a standard deviation of .10 inch. It is normally distributed. What is the probability that a shaft selected at random will be between .55 and .65 inch?

: **T1.6** Refer to Problem T1.5. What is the probability that a shaft size will be greater than .65 inch? What is the probability that a shaft size will be between .53 and .59 inch? What is the probability that a shaft size will be under .45 inch?

: **T1.7** An industrial oven used to cure sand cores for a factory manufacturing engine blocks for small cars is able to maintain fairly constant temperatures. The temperature range of the oven follows a normal distribution, with a mean of 450°F and a standard deviation of 25°F. Kamvar Farahbod, president of the factory, is concerned about the large number of defective cores that have been produced in the last several months. If the oven gets hotter than 475°F, the core is defective. What is the probability that the oven will cause a core to be defective? What is the probability that the temperature of the oven will range from 460° to 470°F?

⋮ T1.8 Bill Hardgrave, production foreman for the Virginia Fruit Company, estimates that the average sales of oranges is 4,700 and the standard deviation is 500 oranges. Sales follow a normal distribution.

a) What is the probability that sales will be greater than 5,500 oranges?
b) What is the probability that sales will be greater than 4,500 oranges?
c) What is the probability that sales will be less than 4,900 oranges?
d) What is the probability that sales will be less than 4,300 oranges?

: T1.9 Lori Becher has been the production manager of Medical Suppliers, Inc. (MSI), for the last 17 years. MSI is a producer of bandages and arm slings. During the last 5 years, the demand for the No-Stick bandage has been fairly constant. On the average, sales have been about 87,000 packages of No-Stick. Lori has reason to believe that the distribution of No-Stick follows a normal curve, with a standard deviation of 4,000 packages. What is the probability sales will be less than 81,000 packages?

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CD TUTORIAL 2

ACCEPTANCE SAMPLING

TUTORIAL OUTLINE

In the Supplement to Chapter 6, *Statistical Process Control*, we briefly introduced the topic of acceptance sampling. **Acceptance sampling** is a form of testing that involves taking random samples of "lots," or batches, of finished products and measuring them against predetermined standards. In this tutorial, we extend our introduction to acceptance sampling by discussing sampling plans, how to build an operating characteristic (OC) curve, and average outgoing quality.

SAMPLING PLANS

A "lot," or batch, of items can be inspected in several ways, including the use of single, double, or sequential sampling.

Single Sampling

Two numbers specify a **single sampling** plan: They are the number of items to be sampled (n) and a prespecified acceptable number of defects (c). If there are fewer or equal defects in the lot than the acceptance number, c, then the whole batch will be accepted. If there are more than c defects, the whole lot will be rejected or subjected to 100% screening.

Double Sampling

Often a lot of items is so good or so bad that we can reach a conclusion about its quality by taking a smaller sample than would have been used in a single sampling plan. If the number of defects in this smaller sample (of size n_1) is less than or equal to some lower limit (c_1), the lot can be accepted. If the number of defects exceeds an upper limit (c_2), the whole lot can be rejected. But if the number of defects in the n_1 sample is between c_1 and c_2, a second sample (of size n_2) is drawn. The cumulative results determine whether to accept or reject the lot. The concept is called **double sampling**.

Sequential Sampling

Multiple sampling is an extension of double sampling, with smaller samples used sequentially until a clear decision can be made. When units are randomly selected from a lot and tested one by one, with the cumulative number of inspected pieces and defects recorded, the process is called **sequential sampling**.

If the cumulative number of defects exceeds an upper limit specified for that sample, the whole lot will be rejected. Or if the cumulative number of rejects is less than or equal to the lower limit, the lot will be accepted. But if the number of defects falls within these two boundaries, we continue to sample units from the lot. It is possible in some sequential plans for the whole lot to be tested, unit by unit, before a conclusion is reached.

Selection of the best sampling approach—single, double, or sequential—depends on the types of products being inspected and their expected quality level. A very low-quality batch of goods, for example, can be identified quickly and more cheaply with sequential sampling. This means that the inspection, which may be costly and/or destructive, can end sooner. On the other hand, in many cases a single sampling plan is easier and simpler for workers to conduct even though the number sampled may be greater than under other plans.

OPERATING CHARACTERISTIC (OC) CURVES

The **operating characteristic (OC) curve** describes how well an acceptance plan discriminates between good and bad lots. A curve pertains to a specific plan, that is, a combination of n (sample size) and c (acceptance level). It is intended to show the probability that the plan will accept lots of various quality levels.

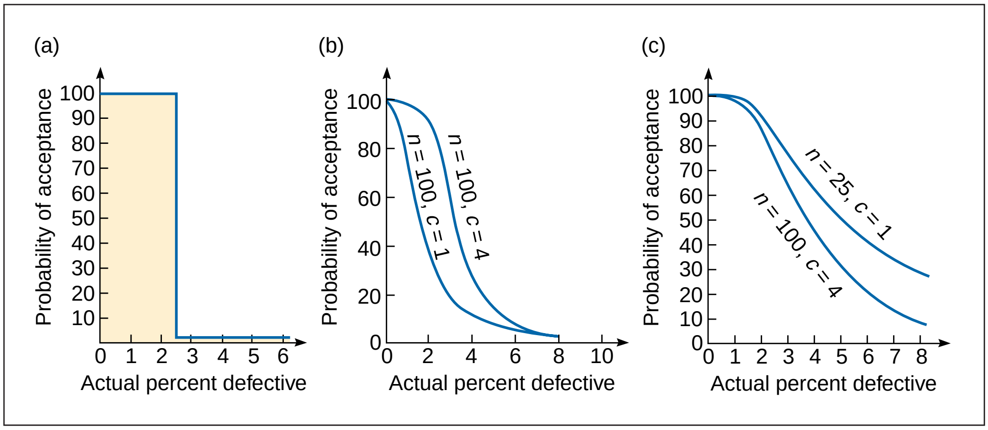

FIGURE T2.1 ■ (a) Perfect Discrimination for Inspection Plan. (b) OC Curves for Two Different Acceptable Levels of Defects ($c = 1$, $c = 4$) for the Same Sample Size ($n = 100$). (c) OC Curves for Two Different Sample Sizes ($n = 25$, $n = 100$) but Same Acceptance Percentages (4%). Larger sample size shows better discrimination.

Naturally, we would prefer a highly discriminating sampling plan and OC curve. If the entire shipment of parts has an unacceptably high level of defects, we hope the sample will reflect that fact with a very high probability (preferably 100%) of rejecting the shipment.

Figure T2.1(a) shows a perfect discrimination plan for a company that wants to reject all lots with more than 2½% defectives and accept all lots with less than 2½% defectives. Unfortunately, the only way to assure 100% acceptance of good lots and 0% acceptance of bad lots is to conduct a full inspection, which is often very costly.

Figure T2.1(b) reveals that no OC curve will be as steplike as the one in Figure T2.1(a); nor will it be discriminating enough to yield 100% error-free inspection. Figure T2.1(b) does indicate, though, that for the same sample size ($n = 100$ in this case), a smaller value of c (of acceptable defects) yields a steeper curve than does a larger value of c. So one way to increase the probability of accepting only good lots and rejecting only bad lots with random sampling is to set very tight acceptance levels.

A second way to develop a steeper, and thereby sounder, OC curve is to increase the sample size. Figure T2.1(c) illustrates that even when the acceptance number is the same proportion of the sample size, a larger value of n will increase the likelihood of accurately measuring the lot's quality. In this figure, both curves use a maximum defect rate of 4% (equal to $4/100 = 1/25$). Yet if you take a straightedge or ruler and carefully examine Figure T2.1(c), you will be able to see that the OC curve for $n = 25$, $c = 1$ rejects more good lots and accepts more bad lots than the second plan. Here are a few measurements to illustrate that point.

When the Actual Percent of Defects in the Lot Is:	Then the Probability (Approximate) of Accepting the Whole Lot Is: For n = 100, c = 4	For n = 25, c = 1
1%	99%	97%
3%	81%	83%
5%	44%	64%
7%	17%	48%

In other words, the probability of accepting a more than satisfactory lot (one with only 1% defects) is 99% for $n = 100$, but only 97% for $n = 25$. Likewise, the chance of accepting a "bad" lot (one with 5% defects) is only 44% for $n = 100$, whereas it is 64% using the smaller sample size.[1] Of course, were it not for the cost of extra inspection, every firm would opt for larger sample sizes.

PRODUCER'S AND CONSUMER'S RISK

In acceptance sampling, two parties are usually involved: the producer of the product and the consumer of the product. In specifying a sampling plan, each party wants to avoid costly mistakes in accepting or rejecting a lot. The producer wants to avoid the mistake of having a good lot rejected (*producer's risk*) because he or she usually must replace the rejected lot. Conversely, the customer or consumer wants to avoid the mistake of accepting a bad lot because defects found in a lot that has already been accepted are usually the responsibility of the customer (*consumer's risk*). The OC curve shows the features of a particular sampling plan, including the risks of making a wrong decision.

To help you understand the theory underlying the use of sampling plans, we will illustrate how an OC curve is constructed statistically.

In attribute sampling, where products are determined to be either good or bad, a binomial distribution is usually employed to build the OC curve. The binomial equation is

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \qquad \textbf{(T2.1)}$$

where n = number of items sampled (called trials)
p = probability that an x (defect) will occur on any one trial
$P(x)$ = probability of exactly x results in n trials

When the sample size (n) is large and the percent defective (p) is small, however, the Poisson distribution can be used as an approximation of the binomial formula. This is convenient because binomial calculations can become quite complex, and because cumulative Poisson tables are readily available. Our Poisson table appears in Appendix II of the text.

In a Poisson approximation of the binomial distribution, the mean of the binomial, which is np, is used as the mean of the Poisson, which is λ; that is,

$$\lambda = np \qquad \textbf{(T2.2)}$$

EXAMPLE T1

A shipment of 2,000 portable battery units for microcomputers is about to be inspected by a Malaysian importer. The Korean manufacturer and the importer have set up a sampling plan in which the α risk is limited to 5% at an acceptable quality level (AQL) of 2% defective, and the β risk is set to 10% at Lot Tolerance Percent Defective (LTPD) = 7% defective. We want to construct the OC curve for the plan of $n = 120$ sample size and an acceptance level of $c \leq 3$ defectives. Both firms want to know if this plan will satisfy their quality and risk requirements.

[1] It bears repeating that sampling always runs the danger of leading to an erroneous conclusion. Let us say in this example that the total population under scrutiny is a load of 1,000 computer chips, of which in reality only 30 (or 3%) are defective. This means that we would want to accept the shipment of chips, because 4% is the allowable defect rate. But if a random sample of $n = 50$ chips were drawn, we could conceivably end up with zero defects and accept that shipment (that is, it is OK) or we could find all 30 defects in the sample. If the latter happened, we could wrongly conclude that the whole population was 60% defective and reject them all.

To solve the problem, we turn to the cumulative Poisson table in Appendix II of the text, whose columns are set up in terms of the acceptance level, c. We are interested only in the $c = 3$ column for this example. The rows in the table are λ ($= np$), which represents the number of defects we would expect to find in each sample.

By varying the percent defectives (p) from .01 (1%) to .08 (8%) and holding the sample size at $n = 120$, we can compute the probability of acceptance of the lot at each chosen level. The values for P (acceptance) calculated in what follows are then plotted to produce the OC curve shown in Figure T2.2.

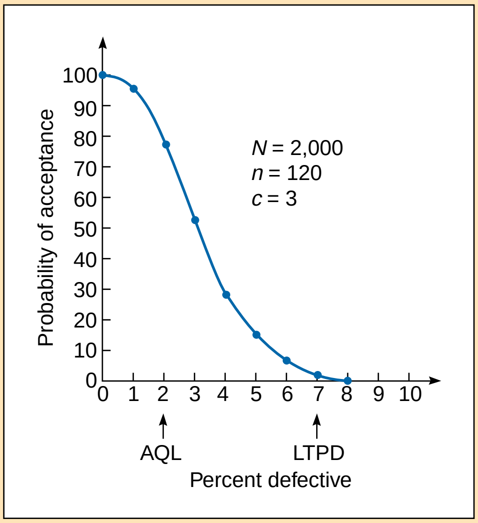

FIGURE T2.2 ■ OC Curve Constructed for Example T1

Selected Values of % Defective	Mean of Poisson, $\lambda = np$	P (Acceptance) from Appendix II
.01	1.20	.966
.02	2.40	.779 ← $1 - \alpha$ at AQL
.03	3.60	.515
.04	4.80	.294
.05	6.00	.151
.06	7.20	.072
.07	8.40	.032* ← β level at LTPD
.08	9.60	.014*

*Interpolated from value.

Now back to the issue of whether this OC curve satisfies the quality and risk needs of the consumer and producer of the batteries. For the AQL of $p = .02 = 2\%$ defects, the P (acceptance) of the lot $= .779$. This yields an α risk of $1 - .779 = .221$, or 22.1%, which exceeds the 5% level desired by the producer. The β risk of .032, or 3.2%, is well under the 10% sought by the consumer. It appears that new calculations are necessary with a larger sample size if the α level is to be lowered.

In Example T1, we set n and c values for a sampling plan and then computed the α and β risks to see if they were within desired levels. Often, organizations instead develop an OC curve for preset values and an AQL and then substitute values of n and c until the plan also satisfies the β and LTPD demands.

AVERAGE OUTGOING QUALITY

In most sampling plans, when a lot is rejected, the entire lot is inspected and all of the defective items are replaced. Use of this replacement technique improves the average outgoing quality in terms of percent defective. In fact, given (1) any sampling plan that replaces all defective items encountered and (2) the true incoming percent defective for the lot, it is possible to determine the **average outgoing quality (AOQ)** in percent defective. The equation for AOQ is

$$\text{AOQ} = \frac{(P_d)(P_a)(N - n)}{N} \qquad \text{(T2.3)}$$

where P_d = true percent defective of the lot
P_a = probability of accepting the lot
N = number of items in the lot
n = number of items in the sample

EXAMPLE T2

The percent defective from an incoming lot in Example T1 is 3%. An OC curve showed the probability of acceptance to be .515. Given a lot size of 2,000 and a sample of 120, what is the average outgoing quality in percent defective?

$$\text{AOQ} = \frac{(P_d)(P_a)(N - n)}{N}$$

$$= \frac{(.03)(.515)(2{,}000 - 120)}{2{,}000} = .015$$

Thus, an acceptance sampling plan changes the quality of the lots in percent defective from 3% to 1.5% on the average. Acceptance sampling significantly increases the quality of the inspected lots.

In most cases, we do not know the value of P_a; we must determine it from the particular sampling plan. The fact that we seldom know the true incoming percent defective presents another problem. In most cases, several different incoming percent defective values are assumed. Then we can determine the average outgoing quality for each value.

EXAMPLE T3

To illustrate the AOQ relationship, let us use the data we developed for the OC curve in Example T1. The lot size in that case was $N = 2{,}000$ and the sample size was $n = 120$. We assume that any defective batteries found during inspection are replaced by good ones. Then using the formula for AOQ given before and the probabilities of acceptance from Example T1, we can develop the following numbers:

P_D	×	P_A	×	$(N - n)/N$	=	AOQ
.01		.966		.94		.009
.02		.779		.94		.015
.03		.515		.94		.015
.04		.294		.94		.011
.05		.151		.94		.007
.06		.072		.94		.004
.07		.032		.94		.002
.08		.014		.94		.001

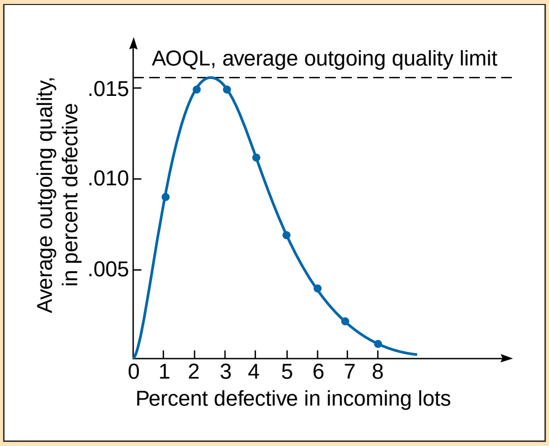

FIGURE T2.3 ■ A Typical AOQ Curve Using Data from Example T3

These numbers are graphed in Figure T2.3 as the average outgoing quality as a function of incoming quality.

Did you notice how AOQ changed for different percent defectives? When the percent defective of the incoming lots is either very high or very low, the percent defective of the outgoing lots is low. AOQ at 1% was .009, and AOQ at 8% was .001. For moderate levels of the incoming percent defective, AOQ is higher: AOQ at 2% to 3% was .015. Thus, AOQ is low for small values of the incoming percent defective. As the incoming percent defective increases, the AOQ increases up to a point. Then, for increasing incoming percent defective, AOQ decreases.

The maximum value on the AOQ curve corresponds to the highest average percent defective or the lowest average quality for the sampling plan. It is called the *average outgoing quality limit (AOQL)*. In Figure T2.3, the AOQL is just over 1.5%, meaning the batteries are about 98.4% good when the incoming quality is between 2% and 3%.

Acceptance sampling is useful for screening incoming lots. When the defective parts are replaced with good parts, acceptance sampling helps to increase the quality of the lots by reducing the outgoing percent defective.

SUMMARY

Acceptance sampling is a major statistical tool of quality control. Sampling plans and operating characteristic (OC) curves facilitate acceptance sampling and provide the manager with tools to evaluate the quality of a production run or shipment.

KEY TERMS

Acceptance sampling *(p. T2-2)*
Single sampling *(p. T2-2)*
Double sampling *(p. T2-2)*
Sequential sampling *(p. T2-2)*
Operating characteristic (OC) curve *(p. T2-2)*
Average outgoing quality (AOQ) *(p. T2-6)*

SOLVED PROBLEM

Solved Problem T2.1

In an acceptance sampling plan developed for lots containing 1,000 units, the sample size n is 85 and c is 3. The percent defective of the incoming lots is 2%, and the probability of acceptance, which was obtained from an OC curve, is 0.64.

What is the average outgoing quality?

Solution

$$\text{AOQ} = \frac{(P_d)(P_a)(N - n)}{N} = \frac{(.02)(.64)(1{,}000 - 85)}{1{,}000} = .012 \text{ or AOQ} = 1.2\%$$

DISCUSSION QUESTIONS

1. Explain the difference between single, double, and sequential sampling.
2. Define AQL and LTPD.
3. What is "average outgoing quality"?
4. What is the AOQL?

PROBLEMS*

- **T2.1** Eighty items are randomly drawn from a lot of 6,000 talking toy animals, and the total lot is accepted if there are $c \leq 2$ defects. Develop an OC curve for this sample plan.
- **T2.2** A load of 200 desk lamps has just arrived at the warehouse of Lighting, Inc. Random samples of $n = 5$ lamps are checked. If more than one lamp is defective, the whole lot is rejected. Set up the OC curve for this plan.
- **T2.3** Develop the AOQ curve for Problem T2.2.
- **T2.4** Each week, Melissa Bryant Ltd. receives a batch of 1,000 popular Swiss watches for its chain of East Coast boutiques. Bryant and the Swiss manufacturer have agreed on the following sampling plan: $\alpha = 5\%$, $\beta = 10\%$, AQL $= 1\%$, LTPD $= 5\%$. Develop the OC curve for a sampling plan of $n = 100$ and $c \leq 2$. Does this plan meet the producer's and consumer's requirements?
- **T2.5** Kristi Conlin's firm in Waco, Texas, has designed an OC curve that shows a ⅔ chance of accepting lots with a true percentage defective of 2%. Lots of 1,000 units are produced at a time, with 100 of each lot sampled randomly. What is the average outgoing quality level?

*Note: P means the problem may be solved with POM for Windows; X means the problem may be solved with Excel OM; and PX means the problem may be solved with POM for Windows and/or Excel OM.

C D T U T O R I A L 3

THE SIMPLEX METHOD OF LINEAR PROGRAMMING

TUTORIAL OUTLINE

Most real-world linear programming problems have more than two variables and thus are too complex for graphical solution. A procedure called the **simplex method** may be used to find the optimal solution to multivariable problems. The simplex method is actually an algorithm (or a set of instructions) with which we examine corner points in a methodical fashion until we arrive at the best solution—highest profit or lowest cost. Computer programs and spreadsheets are available to handle the simplex calculations for you. But you need to know what is involved behind the scenes in order to best understand their valuable outputs.

CONVERTING THE CONSTRAINTS TO EQUATIONS

The first step of the simplex method requires that we convert each inequality constraint in an LP formulation into an equation. Less-than-or-equal-to constraints ($\leq$) can be converted to equations by adding *slack variables*, which represent the amount of an unused resource.

We formulate the Shader Electronics Company's product mix problem as follows, using linear programming:

$$\text{Maximize profit} = \$7X_1 + \$5X_2$$

Subject to LP constraints:

$$2X_1 + 1X_2 \leq 100$$
$$4X_1 + 3X_2 \leq 240$$

where X_1 equals the number of Walkmans produced and X_2 equals the number of Watch-TVs produced.

To convert these inequality constraints to equalities, we add slack variables S_1 and S_2 to the left side of the inequality. The first constraint becomes

$$2X_1 + 1X_2 + S_1 = 100$$

and the second becomes

$$4X_1 + 3X_2 + S_2 = 240$$

To include all variables in each equation (a requirement of the next simplex step), we add slack variables not appearing in each equation with a coefficient of zero. The equations then appear as

$$2X_1 + 1X_2 + 1S_1 + 0S_2 = 100$$
$$4X_1 + 3X_2 + 0S_1 + 1S_2 = 240$$

Because slack variables represent unused resources (such as time on a machine or labor-hours available), they yield no profit, but we must add them to the objective function with zero profit coefficients. Thus, the objective function becomes

$$\text{Maximize profit} = \$7X_1 + \$5X_2 + \$0S_1 + \$0S_2$$

SETTING UP THE FIRST SIMPLEX TABLEAU

To simplify handling the equations and objective function in an LP problem, we place all of the coefficients into a tabular form. We can express the preceding two constraint equations as

Solution Mix	X_1	X_2	S_1	S_2	Quantity (RHS)
S_1	2	1	1	0	100
S_2	4	3	0	1	240

The numbers (2, 1, 1, 0) and (4, 3, 0, 1) represent the coefficients of the first equation and second equation, respectively.

As in the graphical approach, we begin the solution at the origin, where $X_1 = 0$, $X_2 = 0$, and profit $= 0$. The values of the two other variables, S_1 and S_2, then, must be nonzero. Because $2X_1 + 1X_2 + 1S_1 = 100$, we see that $S_1 = 100$. Likewise, $S_2 = 240$. These two slack variables comprise the initial solution mix—as a matter of fact, their values are found in the quantity column across from each variable. Because X_1 and X_2 are not in the solution mix, their initial values are automatically equal to zero.

This initial solution is called a *basic feasible solution* and can be described in vector, or column, form as

$$\begin{bmatrix} X_1 \\ X_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 240 \end{bmatrix}$$

Variables in the solution mix, which is often called the *basis* in LP terminology, are referred to as *basic variables*. In this example, the basic variables are S_1 and S_2. Variables not in the solution mix—or basis—(X_1 and X_2, in this case) are called *non-basic variables*.

Table T3.1 shows the complete initial simplex tableau for Shader Electronics. The terms and rows that you have not seen before are as follows:

C_j: Profit contribution per unit of each variable. C_j applies to both the top row and first column. In the row, it indicates the unit profit for all variables in the LP objective function. In the column, C_j indicates the unit profit for each variable *currently* in the solution mix.

Z_j: In the quantity column, Z_j provides the total contribution (gross profit in this case) of the given solution. In the other columns (under the variables) it represents the gross profit *given up* by adding one unit of this variable into the current solution. The Z_j value for each column is found by multiplying the C_j of the row by the number in that row and jth column and summing.

The calculations for the values of Z_j in Table T3.1 are as follows:

$$Z_j \text{ (for column } X_1) = 0(2) + 0(4) = 0$$

$$Z_j \text{ (for column } X_2) = 0(1) + 0(3) = 0$$

$$Z_j \text{ (for column } S_1) = 0(1) + 0(0) = 0$$

$$Z_j \text{ (for column } S_2) = 0(0) + 0(1) = 0$$

$$Z_j \text{ (for total profit)} = 0(100) + 0(240) = 0$$

$C_j - Z_j$: This number represents the net profit (that is, the profit gained minus the profit given up), which will result from introducing one unit of each product (variable) into the solution. It is not calculated for the quantity column. To compute these numbers, we simply subtract the Z_j total from the C_j value at the very top of each variable's column.

The calculations for the net profit per unit ($C_j - Z_j$) row in this example are as follows:

	Column			
	X_1	X_2	S_1	S_2
C_j for column	\$7	\$5	\$0	\$0
Z_j for column	0	0	0	0
$C_j - Z_j$ for column	\$7	\$5	\$0	\$0

It was obvious to us when we computed a profit of \$0 that this initial solution was not optimal. Examining numbers in the $C_j - Z_j$ row of Table T3.1, we see that total profit can be increased by \$7 for each unit of X_1 (Walkmans) and by \$5 for each unit of X_2 (Watch-TVs) added to the solution mix. A negative number in the $C_j - Z_j$ row would tell us that profits would *decrease* if the corresponding variable were added to the solution mix. An optimal solution is reached in the simplex method when the $C_j - Z_j$ row contains no positive numbers. Such is not the case in our initial tableau.

TABLE T3.1 ■ Completed Initial Simplex Tableau

$C_j \rightarrow$ ↓	SOLUTION MIX	\$7 X_1	\$5 X_2	\$0 S_1	\$0 S_2	QUANTITY (RHS)
\$0	S_1	2	1	1	0	100
\$0	S_2	4	3	0	1	240
	Z_j	\$0	\$0	\$0	\$0	\$0
	$C_j - Z_j$	\$7	\$5	\$0	\$0	(total profit)

SIMPLEX SOLUTION PROCEDURES

Once we have completed an initial tableau, we proceed through a series of five steps to compute all of the numbers we need for the next tableau. The calculations are not difficult, but they are sufficiently complex that the smallest arithmetic error can produce a very wrong answer.

We first list the five steps and then apply them in determining the second and third tableau for the data in the Shader Electronics example.

1. Determine which variable to enter into the solution mix next. Identify the column—hence the variable—with the largest positive number in the $C_j - Z_j$ row of the previous tableau. This step means that we will now be producing some of the product contributing the greatest additional profit per unit.
2. Determine which variable to replace. Because we have just chosen a new variable to enter into the solution mix, we must decide which variable currently in the solution to remove to make room for it. To do so, we divide each amount in the quantity column by the corresponding number in the column selected in step 1. The row with the *smallest nonnegative number* calculated in this fashion will be replaced in the next tableau (this smallest number, by the way, gives the maximum

number of units of the variable that we may place in the solution). This row is often referred to as the **pivot row**, and the column identified in step 1 is called the **pivot column**. The number at the intersection of the pivot row and pivot column is the **pivot number**.

3. Compute new values for the pivot row. To find them, we simply divide every number in the row by the *pivot number*.

4. Compute new values for each remaining row. (In our sample problems there have been only two rows in the LP tableau, but most larger problems have many more rows.) All remaining row(s) are calculated as follows:

$$\begin{pmatrix}\text{New row}\\\text{numbers}\end{pmatrix} = \begin{pmatrix}\text{numbers}\\\text{in old row}\end{pmatrix} - \left[\begin{pmatrix}\text{number in old row}\\\text{above or below}\\\text{pivot number}\end{pmatrix} \times \begin{pmatrix}\text{corresponding number in}\\\text{the new row, i.e., the row}\\\text{replaced in step 3}\end{pmatrix}\right]$$

5. Compute the Z_j and $C_j - Z_j$ rows, as demonstrated in the initial tableau. If all numbers in the $C_j - Z_j$ row are zero or negative, we have found an optimal solution. If this is not the case, we must return to step 1.

All of these computations are best illustrated by using an example. The initial simplex tableau computed in Table T3.1 is repeated below. We can follow the five steps just described to reach an optimal solution to the LP problem.

1st tableau

C_j → ↓	Solution Mix	$7	$5	$0	$0	
		X_1	X_2	S_1	S_2	Quantity
$0	S_1	(2) ← pivot number	1	1	0	100 ← pivot row
$0	S_2	4	3	0	1	240
	Z_j	$0	$0	$0	$0	$0
	$C_j - Z_j$	$7 ↑ pivot column (maximum $C_j - Z_j$ values)	$5	$0	$0	$0

Step 1: Variable X_1 enters the solution next because it has the highest contribution to profit value, $C_j - Z_j$. Its column becomes the pivot column.

Step 2: Divide each number in the quantity column by the corresponding number in the X_1 column: $100/2 = 50$ for the first row and $240/4 = 60$ for the second row. The smaller of these numbers—50—identifies the pivot row, the pivot number, and the variable to be replaced. The pivot row is identified above by an arrow, and the pivot number is circled. Variable X_1 replaces variable S_1 in the solution mix column, as shown in the second tableau.

Step 3: Replace the pivot row by dividing every number in it by the pivot number ($2/2 = 1$, $1/2 = 1/2$, $1/2 = 1/2$, $0/2 = 0$, $100/2 = 50$). This new version of the entire pivot row appears below:

C_j	SOLUTION MIX	X_1	X_2	S_1	S_2	QUANTITY
$7	X_1	1	1/2	1/2	0	50

Step 4: Calculate the new values for the S_2 row.

$$\begin{pmatrix}\text{Number in}\\ \text{new } S_2 \text{ row}\end{pmatrix} = \begin{pmatrix}\text{number in}\\ \text{old } S_2 \text{ row}\end{pmatrix} - \left[\begin{pmatrix}\text{number below}\\ \text{pivot number}\\ \text{in old row}\end{pmatrix} \times \begin{pmatrix}\text{corresponding}\\ \text{number in the}\\ \text{new } X_1 \text{ row}\end{pmatrix}\right]$$

$$\begin{array}{rcrcccr}
0 & = & 4 & - & [(4) & \times & (1)] \\
1 & = & 3 & - & [(4) & \times & (1/2)] \\
-2 & = & 0 & - & [(4) & \times & (1/2)] \\
1 & = & 1 & - & [(4) & \times & (0)] \\
40 & = & 240 & - & [(4) & \times & (50)]
\end{array}$$

C_j	Solution Mix	X_1	X_2	S_1	S_2	Quantity
$7	X_1	1	1/2	1/2	0	50
0	S_2	0	1	−2	1	40

Step 5: Calculate the Z_j and $C_j - Z_j$ rows.

Z_j (for X_1 column) = \$7(1) + 0(0) = \$7 $\qquad C_j - Z_j$ = \$7 − \$7 = 0

Z_j (for X_2 column) = \$7(1/2) + 0(1) = \$7/2 $\qquad C_j - Z_j$ = \$5 − \$7/2 = \$3/2

Z_j (for S_1 column) = \$7(1/2) + 0(−2) = \$7/2 $\qquad C_j - Z_j$ = 0 − \$7/2 = −\$7/2

Z_j (for S_2 column) = \$7(0) + 0(1) = 0 $\qquad C_j - Z_j$ = 0 − 0 = 0

Z_j (for total profit) = \$7(50) + 0(40) = \$350

2nd tableau

$C_j \rightarrow$ ↓		$7	$5	$0	$0	
	Solution Mix	X_1	X_2	S_1	S_2	**Quantity**
$7	X_1	1	1/2	1/2	0	50
$0	S_2	0	(1) ← pivot number	–2	1	40 ← pivot row
	Z_j	$7	$7/2	$7/2	$0	$350 (total profit)
	$C_j - Z_j$	$0	$3/2 ↑ pivot column	$–7/2	$0	

Because not all numbers in the $C_j - Z_j$ row of this latest tableau are zero or negative, the solution (that is, $X_1 = 50$, $S_2 = 40$, $X_2 = 0$, $S_1 = 0$; profit = \$350) is not optimal; we then proceed to a third tableau and repeat the five steps.

Step 1: Variable X_2 enters the solution next because its $C_j - Z_j = 3/2$ is the largest (and only) positive number in the row. Thus, for every unit of X_2 that we start to produce, the objective function will increase in value by \$3/2, or \$1.50.

Step 2: The pivot row becomes the S_2 row because the ratio $40/1 = 40$ is smaller than the ratio $50/(\frac{1}{2}) = 100$.

Step 3: Replace the pivot row by dividing every number in it by the (circled) pivot number. Because every number is divided by one, there is no change.

Step 4: Compute the new values for the X_1 row.

$$\begin{pmatrix}\text{Number in}\\ \text{new } X_1 \text{ row}\end{pmatrix} = \begin{pmatrix}\text{number in}\\ \text{old } X_1 \text{ row}\end{pmatrix} - \left[\begin{pmatrix}\text{number above}\\ \text{pivot number}\end{pmatrix} \times \begin{pmatrix}\text{corresponding}\\ \text{number in the}\\ \text{new } X_2 \text{ row}\end{pmatrix}\right]$$

$$\begin{array}{rcrcrcr}
1 & = & 1 & - & [(1/2) & \times & (0)] \\
0 & = & 1/2 & - & [(1/2) & \times & (1)] \\
3/2 & = & 1/2 & - & [(1/2) & \times & (-2)] \\
-1/2 & = & 0 & - & [(1/2) & \times & (1)] \\
30 & = & 50 & - & [(1/2) & \times & (40)]
\end{array}$$

Step 5: Calculate the Z_j and $C_j - Z_j$ rows.

Z_j (for X_1 column) $= \$7(1) + \$5(0) = \$7$ $\quad C_j - Z_j = \$7 - 7 = \$0$

Z_j (for X_2 column) $= \$7(0) + \$5(1) = \$5$ $\quad C_j - Z_j = \$5 - 5 = \$0$

Z_j (for S_1 column) $= \$7(3/2) + \$5(-2) = \$1/2$ $\quad C_j - Z_j = \$0 - 1/2 = \$-1/2$

Z_j (for S_2 column) $= \$7(-1/2) + \$5(1) = \$3/2$ $\quad C_j - Z_j = \$0 - 3/2 = \$-3/2$

Z_j (for total profit) $= \$7(30) + \$5(40) = \$410$

The results for the third and final tableau are seen in Table T3.2.

Because every number in the third tableau's $C_j - Z_j$ row is zero or negative, we have reached an optimal solution. That solution is: $X_1 = 30$ (Walkmans), and $X_2 = 40$ (Watch-TVs), $S_1 = 0$ (slack in first resource), $S_2 = 0$ (slack in second resource), and profit = \$410.

TABLE T3.2 ■ Third and Final Tableau

$C_j \rightarrow$ ↓	Solution Mix	\$7 X_1	\$5 X_2	\$0 S_1	\$0 S_2	Quantity
\$7	X_1	1	0	3/2	−1/2	30
\$5	X_2	0	1	−2	1	40
	Z_j	\$7	\$5	\$1/2	\$3/2	\$410
	$C_j - Z_j$	\$0	\$0	−\$1/2	−\$3/2	

SUMMARY OF SIMPLEX STEPS FOR MAXIMIZATION PROBLEMS

The steps involved in using the simplex method to help solve an LP problem in which the objective function is to be maximized can be summarized as follows:

1. Choose the variable with the greatest positive $C_j - Z_j$ to enter the solution.
2. Determine the row to be replaced by selecting the one with the smallest (non-negative) ratio of quantity to pivot column.
3. Calculate the new values for the pivot row.
4. Calculate the new values for the other row(s).
5. Calculate the C_j and $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ numbers greater than zero, return to step 1.

ARTIFICIAL AND SURPLUS VARIABLES

Constraints in linear programming problems are seldom all of the "less-than-or-equal-to" ($\leq$) variety seen in the examples thus far. Just as common are "greater-than-or-equal-to" ($\geq$) constraints and equalities. To use the simplex method, each of these also must be converted to a special form. If they are not, the simplex technique is unable to set an initial feasible solution in the first tableau. Example T1 shows how to convert such constraints.

EXAMPLE T1

The following constraints were formulated for an LP problem for the Joyce Cohen Publishing Company. We shall convert each constraint for use in the simplex algorithm.

Constraint 1. $25X_1 + 30X_2 = 900$. To convert an *equality*, we simply add an "artificial" variable (A_1) to the equation:

$$25X_1 + 30X_2 + A_1 = 900$$

An *artificial variable* is a variable that has no physical meaning in terms of a real-world LP problem. It simply allows us to create a basic feasible solution to start the simplex algorithm. An artificial variable is not allowed to appear in the final solution to the problem.

Constraint 2. $5X_1 + 13X_2 + 8X_3 \geq 2{,}100$. To handle $\geq$ constraints, a "surplus" variable (S_1) is first subtracted and then an artificial variable (A_2) is added to form a new equation:

$$5X_1 + 13X_2 + 8X_3 - S_1 + A_2 = 2{,}100$$

A **surplus variable** *does* have a physical meaning—it is the amount over and above a required minimum level set on the right-hand side of a greater-than-or-equal-to constraint.

Whenever an artificial or surplus variable is added to one of the constraints, it must also be included in the other equations and in the problem's objective function, just as was done for slack variables. Each artificial variable is assigned an extremely high cost to ensure that it does not appear in the final solution. Rather than set an actual dollar figure of \$10,000 or \$1 million, however, we simply use the symbol \$M to represent a very large number. Surplus variables, like slack variables, carry a zero cost. Example T2 shows how to figure in such variables.

EXAMPLE T2

The Memphis Chemical Corp. must produce 1,000 lb of a special mixture of phosphate and potassium for a customer. Phosphate costs \$5/lb and potassium costs \$6/lb. No more than 300 lb of phosphate can be used, and at least 150 lb of potassium must be used.

We wish to formulate this as a linear programming problem and to convert the constraints and objective function into the form needed for the simplex algorithm. Let

X_1 = number of pounds of phosphate in the mixture
X_2 = number of pounds of potassium in the mixture

Objective function: minimize cost = $\$5X_1 + \$6X_2$.
Objective function in simplex form:

$$\text{Minimize costs} = \$5X_1 + \$6X_2 + \$0S_1 + \$0S_2 + \$MA_1 + \$MA_2$$

Regular Form	Simplex Form
1st constraint: $1X_1 + 1X_2 = 1{,}000$	$1X_1 + 1X_2 + 1A_1 = 1{,}000$
2nd constraint: $1X_1 \leq 300$	$1X_1 + 1S_1 = 300$
3rd constraint: $1X_2 \geq 150$	$1X_2 - 1S_2 + 1A_2 = 150$

SOLVING MINIMIZATION PROBLEMS

Now that you have worked a few examples of LP problems with the three different types of constraints, you are ready to solve a minimization problem using the simplex algorithm. Minimization problems are quite similar to the maximization problem tackled earlier. The one significant difference involves the $C_j - Z_j$ row. Because our objective is now to minimize costs, the new variable to enter the solution in each tableau (the pivot column) will be the one with the *largest negative* number in the $C_j - Z_j$ row. Thus, we will be choosing the variable that decreases costs the most. In minimization problems, an optimal solution is reached when all numbers in the $C_j - Z_j$ row are *zero* or *positive*—just the opposite from the maximization case. All other simplex steps, as shown, remain the same.

1. Choose the variable with the largest negative $C_j - Z_j$ to enter the solution.
2. Determine the row to be replaced by selecting the one with the smallest (nonnegative) quantity-to-pivot-column ratio.
3. Calculate new values for the pivot row.
4. Calculate new values for the other rows.
5. Calculate the $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ numbers less than zero, return to step 1.

SUMMARY

This tutorial treats a special kind of model, linear programming. LP has proven to be especially useful when trying to make the most effective use of an organization's resources. All LP problems can also be solved with the simplex method, either by computer or by hand. This method is more complex mathematically than graphical LP, but it also produces such valuable economic information as shadow prices. LP is used in a wide variety of business applications.

KEY TERMS

Simplex method *(p. T3-2)*
Pivot row *(p. T3-5)*
Pivot column *(p. T3-5)*
Pivot number *(p. T3-5)*
Surplus variable *(p. T3-8)*

SOLVED PROBLEM

Solved Problem T3.1

Convert the following constraints and objective function into the proper form for use in the simplex method.

Objective function: Minimize cost $= 4X_1 + 1X_2$

Subject to the constraints:

$$3X_1 + X_2 = 3$$
$$4X_1 + 3X_2 \geq 6$$
$$X_1 + 2X_2 \leq 3$$

Solution

$$\text{Minimize cost} = 4X_1 + 1X_2 + 0S_1 + 0S_2 + MA_1 + MA_2$$

Subject to:

$$3X_1 + 1X_2 + 1A_1 = 3$$
$$4X_1 + 3X_2 - 1S_1 + 1A_2 = 6$$
$$1X_1 + 2X_2 + 1S_2 = 3$$

DISCUSSION QUESTIONS

1. Explain the purpose and procedures of the simplex method.
2. How do the graphic and simplex methods of solving linear programming problems differ? In what ways are they the same? Under what circumstances would you prefer to use the graphic approach?
3. What are the simplex rules for selecting the pivot column? The pivot row? The pivot number?
4. A particular linear programming problem has the following objective function:

$$\text{Maximize profit} = \$8X_1 + \$6X_2 + \$12X_3 - \$2X_4$$

Which variable should enter at the second simplex tableau? If the objective function was

$$\text{Minimize cost} = \$2.5X_1 + \$2.9X_2 + \$4.0X_3 + \$7.9X_4$$

which variable would be the best candidate to enter the second tableau?

5. To solve a problem by the simplex method, when are slack variables added?
6. List the steps in a simplex maximization problem.
7. What is a surplus variable? What is an artificial variable?

PROBLEMS*

T3.1 Each coffee table produced by John Alessi Designers nets the firm a profit of $9. Each bookcase yields a $12 profit. Alessi's firm is small and its resources limited. During any given production period (of one week), 10 gallons of varnish and 12 lengths of high-quality redwood are available. Each coffee table requires approximately 1 gallon of varnish and 1 length of redwood. Each bookcase takes 1 gallon of varnish and 2 lengths of wood.

Formulate Alessi's production mix decision as a linear programming problem and solve, using the simplex method. How many tables and bookcases should be produced each week? What will the maximum profit be?

T3.2 **a)** Set up an initial simplex tableau, given the following two constraints and objective function:

$$1X_1 + 4X_2 \le 24$$
$$1X_1 + 2X_2 \le 16$$
$$\text{Maximize profit} = \$3X_1 + \$9X_2$$

You will have to add slack variables.

b) Briefly list the iterative steps necessary to solve the problem in part (a).

c) Determine the next tableau from the one you developed in part (a). Determine whether it is an optimum solution.

d) If necessary, develop another tableau and determine whether it is an optimum solution. Interpret this tableau.

e) Start with the same initial tableau from part (a) but use X_1 as the first pivot column. Continue to iterate it (a total of twice) until you reach an optimum solution.

T3.3 Solve the following linear programming problem graphically. Then set up a simplex tableau and solve the problem, using the simplex method. Indicate the corner points generated at each iteration by the simplex on your graph.

$$\begin{aligned} \text{Maximize profit} &= \$3X_1 + \$5X_2 \\ \text{Subject to:} \quad X_2 &\le 6 \\ 3X_1 + 2X_2 &\le 18 \\ X_1, X_2 &\ge 0 \end{aligned}$$

*Note: **P** means the problem may be solved with POM for Windows; X means the problem may be solved with Excel OM; and PX means the problem may be solved with POM for Windows and/or Excel OM.

: T3.4 Solve the following linear programming problem, first graphically and then by simplex algorithm.

$$\begin{aligned} \text{Maximize cost} &= 4X_1 + 5X_2 \\ \text{Subject to:}\quad X_1 + 2X_2 &\geq 80 \\ 3X_1 + X_2 &\geq 75 \\ X_1, X_2 &\geq 0 \end{aligned}$$

What are the values of the basic variables at each iteration? Which are the non-basic variables at each iteration?

: T3.5 Barrow Distributors packages and distributes industrial supplies. A standard shipment can be packaged in a Class A container, a Class K container, or a Class T container. A single Class A container yields a profit of \$8; a Class K container, a profit of \$6; and a Class T container, a profit of \$14. Each shipment prepared requires a certain amount of packing material and a certain amount of time.

Resources Needed per Standard Shipment

CLASS OF CONTAINER	PACKING MATERIAL (POUNDS)	PACKING TIME (HOURS)
A	2	2
K	1	6
T	3	4
Total amount of resource available each week	120 pounds	240 hours

Joe Barrow, head of the firm, must decide the optimal number of each class of container to pack each week. He is bound by the previously mentioned resource restrictions but also decides that he must keep his six full-time packers employed all 240 hours (6 workers × 40 hours) each week.

Formulate and solve this problem, using the simplex method.

: T3.6 Set up a complete initial tableau for the data (repeated below) that were first presented in Solved Problem T3.1.

$$\begin{aligned} \text{Minimize cost} = {} & 4X_1 + 1X_2 + 0S_1 + 0S_2 + MA_1 + MA_2 \\ \text{Subject to:}\quad & 3X_1 + 1X_2 + 1A_1 = 3 \\ & 4X_1 + 3X_2 - 1S_1 + 1A_2 = 6 \\ & 1X_1 + 2X_2 + 1S_2 = 3 \end{aligned}$$

a) Which variable will enter the solution next?

b) Which variable will leave the solution?

: T3.7 Solve Problem T3.6 for the optimal solution, using the simplex method.

CD TUTORIAL 4

The MODI and VAM Methods of Solving Transportation Problems

TUTORIAL OUTLINE

This tutorial deals with two techniques for solving transportation problems: the MODI method and Vogel's Approximation Method (VAM).

MODI METHOD

The MODI (*modified distribution*) method allows us to compute improvement indices quickly for each unused square without drawing all of the closed paths. Because of this, it can often provide considerable time savings over other methods for solving transportation problems.

MODI provides a new means of finding the unused route with the largest negative improvement index. Once the largest index is identified, we are required to trace only one closed path. This path helps determine the maximum number of units that can be shipped via the best unused route.

How to Use the MODI Method

In applying the MODI method, we begin with an initial solution obtained by using the northwest corner rule or any other rule. But now we must compute a value for each row (call the values R_1, R_2, R_3 if there are three rows) and for each column (K_1, K_2, K_3) in the transportation table. In general, we let

R_i = value assigned to row i

K_j = value assigned to column j

C_{ij} = cost in square ij (cost of shipping from source i to destination j)

The MODI method then requires five steps:

1. To compute the values for each row and column, set

$$R_i + K_j = C_{ij}$$

but *only for those squares that are currently used or occupied*. For example, if the square at the intersection of row 2 and column 1 is occupied, we set $R_2 + K_1 = C_{21}$.

2. After all equations have been written, set $R_1 = 0$.

3. Solve the system of equations for all R and K values.

4. Compute the improvement index for each unused square by the formula improvement index $(I_{ij}) = C_{ij} - R_i - K_j$

5. Select the largest negative index and proceed to solve the problem as you did using the stepping-stone method.

Solving the Arizona Plumbing Problem with MODI

Let us try out these rules on the Arizona Plumbing problem. The initial northwest corner solution is shown in Table T4.1. MODI will be used to compute an improvement index for each unused square. Note that the only change in the transportation table is the border labeling the R_is (rows) and K_js (columns).

We first set up an equation for each occupied square:

(1) $R_1 + K_1 = 5$

(2) $R_2 + K_1 = 8$

TABLE T4.1 ■ **Initial Solution to Arizona Plumbing Problem in the MODI Format**

	K_j	K_1	K_2	K_3	
R_i	FROM / TO	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
R_1	DES MOINES	5 100	4	3	100
R_2	EVANSVILLE	8 200	4 100	3	300
R_3	FORT LAUDERDALE	9	7 100	5 200	300
	WAREHOUSE REQUIREMENTS	300	200	200	700

(3) $R_2 + K_2 = 4$

(4) $R_3 + K_2 = 7$

(5) $R_3 + K_3 = 5$

Letting $R_1 = 0$, we can easily solve, step by step, for K_1, R_2, K_2, R_3, and K_3.

(1) $R_1 + K_1 = 5$

$0 + K_1 = 5 \qquad K_1 = 5$

(2) $R_2 + K_1 = 8$

$R_2 + 5 = 8 \qquad R_2 = 3$

(3) $R_2 + K_2 = 4$

$3 + K_2 = 4 \qquad K_2 = 1$

(4) $R_3 + K_2 = 7$

$R_2 + 1 = 7 \qquad R_3 = 6$

(5) $R_3 + K_3 = 5$

$6 + K_3 = 5 \qquad K_3 = -1$

You can observe that these R and K values will not always be positive; it is common for zero and negative values to occur as well. After solving for the Rs and Ks in a few practice problems, you may become so proficient that the calculations can be done in your head instead of by writing the equations out.

The next step is to compute the improvement index for each unused cell. That formula is

$$\text{improvement index} = I_{ij} = C_{ij} - R_i - K_j$$

We have:

Des Moines–Boston index $= I_{DB}$ (or I_{12}) $= C_{12} - R_1 - K_2 = 4 - 0 - 1$
$= +\$3$

Des Moines–Cleveland index $= I_{DC}$ (or I_{13}) $= C_{13} - R_1 - K_3 = 3 - 0 - (-1)$
$= +\$4$

Evansville–Cleveland index $= I_{EC}$ (or I_{23}) $= C_{23} - R_2 - K_3 = 3 - 3 - (-1)$
$= +\$1$

Fort Lauderdale–Albuquerque index $= I_{FA}$ (or I_{31}) $= C_{31} - R_3 - K_1 = 9 - 6 - 5$
$= -\$2$

Because one of the indices is negative, the current solution is not optimal. Now it is necessary to trace only the one closed path, for Fort Lauderdale–Albuquerque, in order to proceed with the solution procedures.

The steps we follow to develop an improved solution after the improvement indices have been computed are outlined briefly:

1. Beginning at the square with the best improvement index (Fort Lauderdale–Albuquerque), trace a closed path back to the original square via squares that are currently being used.
2. Beginning with a plus (+) sign at the unused square, place alternate minus (−) signs and plus signs on each corner square of the closed path just traced.
3. Select the smallest quantity found in those squares containing minus signs. *Add* that number to all squares on the closed path with plus signs; *subtract* the number from all squares assigned minus signs.
4. Compute new improvement indices for this new solution using the MODI method.

Following this procedure, the second and third solutions to the Arizona Plumbing Corporation problem can be found. See Tables T4.2 and T4.3. With each new MODI solu-

FROM \ TO	A	B	C	FACTORY
D	100 ($5)	($4)	($3)	100
E	100 ($8)	200 ($4)	($3)	300
F	100 ($9)	($7)	200 ($5)	300
WAREHOUSE	300	200	200	700

TABLE T4.2 ■ **Second Solution to the Arizona Plumbing Problem**

TABLE T4.3 ■ Third and Optimal Solution to Arizona Plumbing Problem

FROM \ TO	*A*	*B*	*C*	FACTORY
D	100 ($5)	($4)	($3)	100
E	($8)	200 ($4)	100 ($3)	300
F	200 ($9)	($7)	100 ($5)	300
WAREHOUSE	300	200	200	700

tion, we must recalculate the *R* and *K* values. These values then are used to compute new improvement indices in order to determine whether further shipping cost reduction is possible.

VOGEL'S APPROXIMATION METHOD: ANOTHER WAY TO FIND AN INITIAL SOLUTION

In addition to the northwest corner and intuitive lowest-cost methods of setting an initial solution to transportation problems, we introduce one other important technique—*Vogel's approximation method* (VAM). VAM is not quite as simple as the northwest corner approach, but it facilitates a very good initial solution—as a matter of fact, one that is often the *optimal* solution.

Vogel's approximation method tackles the problem of finding a good initial solution by taking into account the costs associated with each route alternative. This is something that the northwest corner rule did not do. To apply the VAM, we first compute for each row and column the penalty faced if we should ship over the *second best* route instead of the *least-cost* route.

The six steps involved in determining an initial VAM solution are illustrated on the Arizona Plumbing Corporation data. We begin with Table T4.4.

VAM Step 1: For each row and column of the transportation table, find the difference between the two lowest unit shipping costs. These numbers represent the difference between the distribution cost on the *best* route in the row or column and the *second best* route in the row or column. (This is the *opportunity cost* of not using the best route.)

Step 1 has been done in Table T4.5. The numbers at the heads of the columns and to the right of the rows represent these differences. For example, in row *E* the three transportation costs are $8, $4, and $3. The two lowest costs are $4 and $3; their difference is $1.

VAM Step 2: Identify the row or column with the greatest opportunity cost, or difference. In the case of Table T4.5, the row or column selected is column *A*, with a difference of 3.

TABLE T4.4 ■ Transportation Table for Arizona Plumbing Corporation

FROM \ TO	Warehouse at Albuquerque	Warehouse at Boston	Warehouse at Cleveland	Factory Capacity
Des Moines factory	$5	$4	$3	100
Evansville factory	$8	$4	$3	300
Fort Lauderdale factory	$9	$7	$5	300
Warehouse requirements	300	200	200	700

Des Moines capacity constraint

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

Cleveland warehouse demand

Total demand and total supply

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

VAM Step 3: Assign as many units as possible to the lowest cost square in the row or column selected.

Step 3 has been done in Table T4.6. Under Column *A*, the lowest-cost route is *D–A* (with a cost of $5), and 100 units have been assigned to that square. No more were placed in the square because doing so would exceed *D*'s availability.

VAM Step 4. Eliminate any row or column that has just been completely satisfied by the assignment just made. This can be done by placing *X*s in each appropriate square.

FROM \ TO	ALBUQUERQUE *A*	BOSTON *B*	CLEVELAND *C*	TOTAL AVAILABLE	
	3	0	0		
DES MOINES *D*	5	4	3	100	1
EVANSVILLE *E*	8	4	3	300	1
FORT LAUDERDALE *F*	9	7	5	300	2
TOTAL REQUIRED	300	200	200	700	

TABLE T4.5 ■ Transportation Table with VAM Row and Column Differences Shown

TABLE T4.6 ■ VAM Assignment with *D*'s Requirements Satisfied

	~~3~~ 1	~~0~~ 3	~~0~~ 2		
FROM \ TO	***A***	***B***	***C***	**TOTAL AVAILABLE**	
D	[5] 100	[4] X	[3] X	100	~~1~~
E	[8]	[4]	[3]	300	1
F	[9]	[7]	[5]	300	2
TOTAL REQUIRED	300	200	200	700	

Step 4 has been done in Table T4.6 *D* row. No future assignments will be made to the *D–B* or *D–C* routes.

VAM Step 5: Recompute the cost differences for the transportation table, omitting rows or columns crossed out in the preceding step.

This is also shown in Table T4.6. *A*'s, *B*'s, and *C*'s differences each change. *D*'s row is eliminated, and *E*'s and *F*'s differences remain the same as in Table T4.5.

VAM Step 6: Return to step 2 and repeat the steps until an initial feasible solution has been obtained.

	~~3~~ 1	~~0~~ 3	~~0~~ 2		
FROM \ TO	***A***	***B***	***C***	**TOTAL AVAILABLE**	
D	[5] 100	[4] X	[3] X	100	~~1~~
E	[8]	[4] 200	[3]	300	~~1~~ 5
F	[9]	[7] X	[5]	300	~~2~~ 4
TOTAL REQUIRED	300	200	200	700	

TABLE T4.7 ■ Second VAM Assignment with *B*'s Requirements Satisfied

TABLE T4.8 ■ Third VAM Assignment with *C*'s Requirements Satisfied

FROM \ TO	*A*	*B*	*C*	TOTAL AVAILABLE
D	5 100	4 X	3 X	100
E	8 X	4 200	3 100	300
F	9	7 X	5	300
TOTAL REQUIRED	300	200	200	700

In our case, column *B* now has the greatest difference, which is 3. We assign 200 units to the lowest-cost square in column *B* that has not been crossed out. This is seen to be *E–B*. Since *B*'s requirements have now been met, we place an X in the *F–B* square to eliminate it. Differences are once again recomputed. This process is summarized in Table T4.7.

The greatest difference is now in row *E*. Hence, we shall assign as many units as possible to the lowest-cost square in row *E*, that is, *E–C* with a cost of $3. The maximum assignment of 100 units depletes the remaining availability at *E*. The square *E–A* may therefore be crossed out. This is illustrated in Table T4.8.

The final two allocations, at *F–A* and *F–C*, may be made by inspecting supply restrictions (in the rows) and demand requirements (in the columns). We see that an assignment of 200 units to *F–A* and 100 units to *F–C* completes the table (see Table T4.9).

FROM \ TO	*A*	*B*	*C*	TOTAL AVAILABLE
D	5 100	4 X	3 X	100
E	8 X	4 200	3 100	300
F	9 200	7 X	5 100	300
TOTAL REQUIRED	300	200	200	700

TABLE T4.9 ■ Final Assignments to Balance Column and Row Requirements

The cost of this VAM assignment is = (100 units × \$5) + (200 units × \$4) + (100 units × \$3) + (200 units × \$9) + (100 units × \$5) = \$3,900.

It is worth noting that the use of Vogel's approximation method on the Arizona Plumbing Corporation data produces the optimal solution to this problem. Even though VAM takes many more calculations to find an initial solution than does the northwest corner rule, it almost always produces a much better initial solution. Hence VAM tends to minimize the total number of computations needed to reach an optimal solution.

DISCUSSION QUESTIONS

1. Why does Vogel's approximation method provide a good initial feasible solution? Could the northwest corner rule ever provide an initial solution with as low a cost?

2. How do the MODI and stepping-stone methods differ?

PROBLEMS*

P : **T4.1** The Hardrock Concrete Company has plants in three locations and is currently working on three major construction projects, each located at a different site. The shipping cost per truckload of concrete, daily plant capacities, and daily project requirements are provided in the accompanying table.

From \ To	Project *A*	Project *B*	Project *C*	Plant Capacities
Plant 1	\$10	\$ 4	\$11	70
Plant 2	12	5	8	50
Plant 3	9	7	6	30
Project Requirements	40	50	60	150

(a) Formulate an initial feasible solution to Hardrock's transportation problem using VAM.
(b) Then solve using the MODI method.
(c) Was the initial solution optimal?

P : **T4.2** Hardrock Concrete's owner has decided to increase the capacity at his smallest plant (see Problem T4.1). Instead of producing 30 loads of concrete per day at plant 3, that plant's capacity is doubled to 60 loads. Find the new optimal solution using VAM and MODI. How has changing the third plant's capacity altered the optimal shipping assignment?

P · **T4.3** The Saussy Lumber Company ships pine flooring to three building supply houses from its mills in Pineville, Oak Ridge, and Mapletown. Determine the best transportation schedule for the data given in the accompanying table. Use the northwest corner rule and the MODI method.

*Note: **P** means the problem may be solved with POM for Windows; means the problem may be solved with Excel OM; and **P** means the problem may be solved with POM for Windows and/or Excel OM.

TO / FROM	SUPPLY HOUSE 1	SUPPLY HOUSE 2	SUPPLY HOUSE 3	MILL CAPACITY (TONS)
PINEVILLE	$3	$3	$2	25
OAK RIDGE	4	2	3	40
MAPLETOWN	3	2	3	30
SUPPLY HOUSE DEMAND (TONS)	30	30	35	95

: **T4.4** The Krampf Lines Railway Company specializes in coal handling. On Friday, April 13, Krampf had empty cars at the following towns in the quantities indicated:

Town	Supply of Cars
Morgantown	35
Youngstown	60
Pittsburgh	25

By Monday, April 16, the following towns will need coal cars:

Town	Demand for Cars
Coal Valley	30
Coaltown	45
Coal Junction	25
Coalsburg	20

Using a railway city-to-city distance chart, the dispatcher constructs a mileage table for the preceding towns. The result is

	To			
From	Coal Valley	Coaltown	Coal Junction	Coalsburg
Morgantown	50	30	60	70
Youngstown	20	80	10	90
Pittsburgh	100	40	80	30

Minimizing total miles over which cars are moved to new locations, compute the best shipment of coal cars. Use the northwest corner rule and the MODI method.

: **T4.5** The Jessie Cohen Clothing Group owns factories in three towns (*W*, *Y*, and *Z*) that distribute to three Cohen retail dress shops (in *A*, *B*, and *C*). Factory availabilities, projected store demands, and unit shipping costs are summarized in the table that follows:

FROM \ TO	A	B	C	FACTORY AVAILABILITY
W	4	3	3	35
X	6	7	6	50
Y	8	2	5	50
STORE DEMAND	30	65	40	135

Use Vogel's approximation method to find an initial feasible solution to this transportation problem. Is your VAM solution optimal?

: T4.6 The state of Missouri has three major power-generating companies (*A*, *B*, and *C*). During the months of peak demand, the Missouri Power Authority authorizes these companies to pool their excess supply and to distribute it to smaller independent power companies that do not have generators large enough to handle the demand. Excess supply is distributed on the basis of cost per kilowatt hour transmitted. The accompanying table shows the demand and supply in millions of kilowatt hours and the costs per kilowatt hour of transmitting electric power to four small companies in cities *W*, *X*, *Y*, and *Z*.

From \ To	W	X	Y	Z	Excess Supply
A	12¢	4¢	9¢	5¢	55
B	8¢	1¢	6¢	6¢	45
C	1¢	12¢	4¢	7¢	30
Unfilled Power Demand	40	20	50	20	

Use Vogel's approximation method to find an initial transmission assignment of the excess power supply. Then apply the MODI technique to find the least-cost distribution system.